SUPPLEMENT TO

MERCHANT'S ELEMENTARY MECHANICS

DEALING WITH THE PHENOMENA OF

SURFACE TENSION AND THE FLOW OF LIQUIDS

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PREFACE.

This work has been prepared in answer to a demand for information regarding the phenomena of Surface Tension and the Flow of Liquids, subjects which are treated very briefly, if at all, in many books on Physics.

It is now published separately, but in future editions of Merchant's "Elementary Mechanics" it will be incorporated as two additional chapters, and the pages have been numbered with that object in view.

The author wishes to acknowledge valuable assistance received from Professor F. B. Kenrick of the Department of Chemistry, University of Toronto, and Mr. G. A. Cornish, B.A., Chief Science Master in the University Schools, Toronto.

C. A. CHANT.

TORONTO, *January, 1912.*

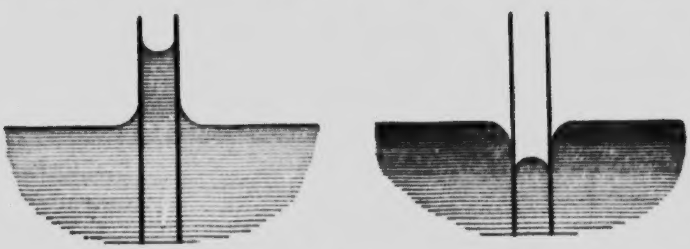
CHAPTER XVIII.

SURFACE TENSION.

1. Change of Level by Surface Tension.

Experiment 1.

Hold a small glass tube upright in water and observe the behaviour of the water within the tube. It will be seen to rise above the level outside. Note also that the water curves upward where it touches the glass (Fig. 116). This effect can



Water. Mercury.

FIG. 116.

be observed more easily if a little colouring matter (magenta, for example) is added to the water. If mercury is used instead of water, the liquid within the tube takes a lower level than that outside, and where it touches the glass it curves downward instead of upward.

Now lift the glass tube out of the water, and note that some of the liquid clings to it. Do the same with the mercury; you will see that none at all adheres. Water is said to wet the glass; mercury does not. In these experiments the glass should be perfectly clean.

Again, if water is sprinkled on a very dusty floor it usually forms in drops which persist for some time. Drop a little

mercury on the floor; it breaks into a multitude of shining globules which retain their rounded forms permanently.

Now there is a very important principle in mechanics which states that **the potential energy of a system of bodies always tends to a minimum.** A body if unsupported falls to the earth, thus parting with its potential energy. The foundations of a house must be strong enough to support the superstructure or it will crumble to the ground. A compressed or a stretched spring or a bent bow endeavours to give up its potential energy and to return to its natural unstrained form. Water continually seeks a lower level. If placed in a vessel it tries to spread out and take up as low a position as possible. In doing so its surface becomes a horizontal plane, and in a series of connecting tubes it reaches the same level in each (see p. 184, Exp. 4).

The experiments with the small tube just described would, at first sight, appear to be inconsistent with this principle. On examination, however, we shall find that such is not the case, but that, on the other hand, they illustrate it in a new and beautiful manner.

2. Tension in the Surface.

Observations upon liquids exposed to air strongly suggest that they are enclosed in a thin skin or membrane, which continually tends to contract. The globules of water and of mercury illustrate this.

Experiment 1.

Fill a wine-glass or a small tumbler brimful of water, and then carefully drop into it coins, buttons or other bits of metal. The water slowly rises above the top of the glass, appearing to be restrained within a skin which clings at its edges to the glass.

The surface becomes more and more convex until at last the skin breaks and the water runs over the edge.

Fine iron dust or gold-leaf rests quietly on the surface of water, though the former is 7, and the latter 19, times as dense as the water. In both cases they are not heavy enough to break through the skin on the surface. Remember, however, that this skin is made of liquid, though it is reasonable to suppose that the constitution of the surface layer is slightly different from that of the rest of the liquid.

Experiment 2.

Put water in a beaker and then carefully pour alcohol on top of it. About 40 per cent. of water to 60 per cent. of alcohol is best, but there may be considerable variation from this proportion. Now introduce olive oil into it by means of a pipette* (Fig. 117). If it is of the same density it will neither sink nor rise on account of gravity. It assumes a spherical form **as though an enveloping skin was trying to compress the oil into a smaller space.** For a given volume a sphere has less surface area than a body of any other form, and in assuming this shape the potential energy of the surface tends to a minimum.

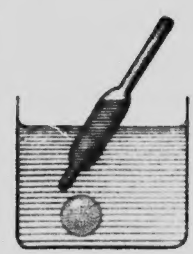


FIG. 117.

Under ordinary circumstances it is impossible to observe the effect of surface tension as exhibited in the successive stages in the formation of a drop of water. It grows rapidly, and its weight causes it to break away while it is still small.

A good way to study the formation of a drop is as follows†:—

*Full instructions for performing this experiment in the most satisfactory way are given in "Soap Bubbles," by C. V. Boys, page 141.

†Devised by Chas. R. Darling, *Nature*, Vol. 83, p. 37, 1910.

Experiment 3.

Aniline is an oily liquid which at ordinary temperatures is denser than water. When poured into water it does not mix with it, but falls to the bottom, and the colour assumed by the aniline renders the surface between the water and the aniline clearly visible at a considerable distance. However when heated above 80° C. it rises to the surface of the water.

Into a beaker about 9 inches high and $4\frac{1}{2}$ inches in diameter pour water to the depth of about 7 inches. Then add about 80 c.c. of aniline. Place the beaker above a burner and heat gently until a temperature of about 80° is reached.

The hot aniline now rises to the surface, spreads out, and, coming in contact with the air, is cooled and collects in the form of a drop, an inch or more in diameter, hanging down from the mass at the surface. The process is so slow that it can be studied in detail. As the drop grows in size a neck is formed, which after a while gets thinner at two places; and when it breaks away the large drop is followed by a small one. If the temperature is maintained at about 80° the drops will continue to be formed.

Observe the oscillations in the form of the drop as it descends.

Various stages in the development of a drop of water are illustrated in Fig. 118.

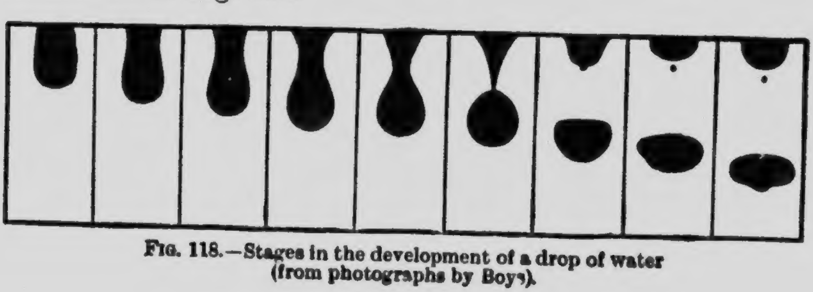


FIG. 118.—Stages in the development of a drop of water (from photographs by Boys).

Experiment 4.

Again, take a ring of wire about 2 inches in diameter, with a handle on it (Fig. 119). To two points on the ring tie a fine thread with a loop in it. Dip the ring in a soap solution,* and obtain a film across it with the loop resting on the film. This film is a thin layer of water bounded by two surfaces, the soap making it more permanent. Now puncture the film within the loop. The film which is left contracts, becomes as small as possible and thus draws the loop into a circle, since the area of a circle is greater than that of any other surface having an equal perimeter. Note again how the energy of the film tends to a minimum.

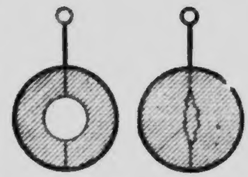


Fig. 119.

The surface thus acts like a stretched sheet of india-rubber, and exerts a tension of the same kind. But there is a difference between them. The tension in the sheet of rubber depends on the amount of stretching, and may be greater in one direction than in another; whereas the tension in the soap-film remains the same however much the film is extended, and the tension at any point is the same in all directions along the film.

Experiment 5.†

Dip the upper edge of a rectangular glass vessel (a projection tank) into melted paraffin wax, and then carefully pour in water until its surface curves over at the top of the tank (Fig. 120).

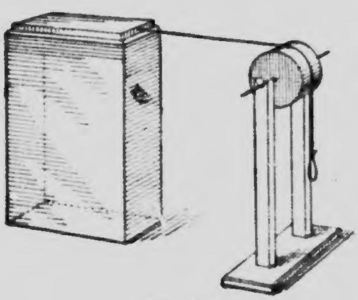


Fig. 120.

The surface is rounded at the edges and its tension causes it to shrink to as small an area as possible.

On the surface lay a thin layer of cork, and let a thread,

* See method of preparation, page 266.

† This experiment is due to Prof. Kenrick.

attached to one end of this, pass over a little pulley (made of a pill-box with a needle for axis).

On pulling the string gently the surface is stretched to a greater area, and on letting go it springs back to its original form.

By adding bits of bent wire to the loop on the end of the thread the stretching of the surface can easily be observed, and can be projected on the screen.

3. Surface Energy.

To inflate a rubber balloon or a bicycle tire, or to blow a soap-bubble requires an expenditure of work; and when these bodies contract they exert a force and thus can do work.

Experiment 1.

The fact that a soap-film will contract and exert a force can be well shown as follows: Bend a wire into a rectangular shape (Fig. 121) and dip it into a soap solution. On taking it out it is covered with a film. Hold it horizontal and across it lay a thin straight wire *mn*; then puncture the side Q of the film. The two surfaces of the film P which is left exert a tension on the wire in the direction shown by the arrow, and draw the wire over to the end *ab*, thus reducing the area of the film to as small dimensions as possible.

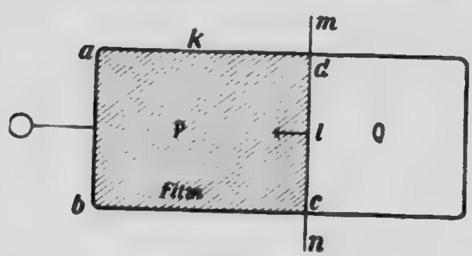


Fig. 121.

By an experiment somewhat similar to this the magnitude of the surface tension can be roughly determined (see Section 7, below).

It is evident that the greater the width *cd* of the rectangle, the greater will be the entire force drawing the wire in the direction of the arrow, *i.e.*, at right angles to the axis of the wire.

Let the width *cd* of the rectangle be l cm., and let the tension exerted by each surface of the film, on each cm. of the wire, be T dynes. Then the entire tension exerted upon the wire by the two surfaces of the film = $2Tl$ dynes. If the length *ad* of the film is k cm., the work which the film P can do in contracting = $2Tlk$ ergs.

Just as we say that a bent bow or a stretched sheet of rubber possesses potential energy, so we can say that the film possesses potential energy, and its amount is equal to the work which it can do in contracting, that is, $2Tlk$ ergs. But its area = $2lk$ sq. cm. Hence the potential energy per sq. cm. = $2Tlk \div 2lk = T$ ergs.

Again, if one takes hold of the wire and moves it to the right (Fig. 121) a distance x cm., thus increasing the area of the film by $2lx$ sq. cm., the work which one does is $2Tlx$ ergs, and the work done per sq. cm. = T ergs.

Hence we have the relation:—**The measure of the surface tension of a liquid is equal to the measure of its potential energy per sq. cm. of the surface; or it is equal to the measure of the work done in enlarging the surface of the liquid one unit of area.**

Remember, also, that the surface tension is measured in dynes across a linear centimetre.

The quantity T is usually called the **Coefficient of Capillarity.**

The question of surface tension arose chiefly through the consideration of the rise of liquids in capillary tubes, *i.e.*, tubes so fine as to admit only a hair (Latin, *capillus*, a hair); but the subject of surface tension is a very broad one with numerous applications. Hence, it is better to use the name **surface tension** than the name **capillarity**, by which it is sometimes known.

4. Angle of Contact or Capillary Angle.

We have seen that when a plate of glass is held vertically in water, the liquid, where it touches the glass, is drawn up above the level of the general surface (*a*, Fig. 122).

FIG. 122.

The angle which the tangent to the liquid surface where it meets the surface of the solid makes with the common surface of the liquid and the solid is called the **angle of contact** or the **capillary angle** (Fig. 122).

The size of this angle depends on the third medium, above the liquid. Thus if oil is used instead of air the angle is much altered. It also depends very materially on the condition of the surfaces. The slightest contamination on the surface of water or on the solid will alter the angle considerably. Figure 122*a* illustrates the usual condition for water and glass. With perfectly clean

water and glass the angle of contact BAC is very small, probably zero, but with slight contamination it may reach 90°, *i.e.*, it does not rise on the surface of the glass at all. Figure 121*b* illustrates the effect with mercury and glass. Here the angle of contact is obtuse, varying from 129° to 143°.

5. Rise of a Liquid in a Tube.

Consider a tube held vertically in a liquid which wets it. The liquid rises on the outside slightly, but on the inside to a considerable height (Fig. 123).

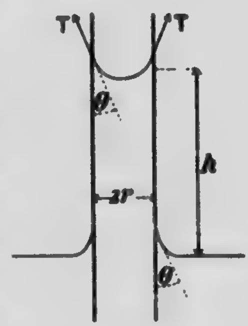


Fig 123.

The phenomenon is "explained" by stating that the attraction of the molecules of the liquid for those of the glass is greater than the attraction of the molecules of the liquid for each other. The surface of the liquid meets the glass along an inner circumference of the tube, and the attraction exerted, across this line, between the surface molecules of the liquid and those of the glass, is sufficient to support the raised column.

Let T denote the surface tension in dynes per cm.
r " " radius of tube in cm.
h " " mean height of column in cm.
ρ " " density of the liquid in gm. per c.c.
θ " " angle of contact.

The force T acts in a direction making an angle θ with the vertical; hence its component in the vertical is $T \cos \theta$.

The surface of the liquid pulls the inner surface of the tube inwards and downwards, acting in the direction of the tangent to the liquid surface where it touches the tube, and the reaction of the tube lifts the liquid upward.

The length of the line of contact of liquid and inner surface of the tube $= 2\pi r$ cm., and hence the total force upward in the direction of the axis of the tube

$$= 2\pi r\text{T} \cos \theta \text{ dynes.}$$

This balances the weight of the raised column of liquid, which $= \pi r^2 h\rho g$ dynes.

Equating the total force upward to the total force downward, we have

$$\pi r^2 h\rho g = 2\pi r \text{ T} \cos \theta,$$

$$\text{and T} = \frac{h\rho r g}{2 \cos \theta}, \text{ or } h = \frac{2\text{T} \cos \theta}{\rho r g}.$$

From this we see that $h \propto \frac{1}{r}$, or **the height to which the liquid is drawn up is inversely proportional to the radius of the tube.** With a very small tube the rise of the liquid may be considerable.

In a glass tube of radius 1 mm. the water rises about 1.4 cm. Hence in one of radius $\frac{1}{1000}$ mm. the rise would be 14 metres. It has been surmised that the distribution of sap in plants is partially due to capillary action, but this will not account for the *rate* at which water rises in trees.

The tube of a barometer should be large, otherwise a correction for capillarity is necessary. If the tube has a diameter of 2 mm. the mercury is depressed 4.6 mm., but

if it is 2 cm. (about 0.8 inch) or greater the correction for depression is so small that it may be neglected.

6. Attraction and Repulsion between Bodies on the Surface of Water.

It has often been noticed that bubbles, small sticks and straws floating on still water appear to attract each other. They gather in groups or become attached to the edge of the containing vessel. This effect can be easily illustrated by means of two discs sliced off a cork, placed on the surface of the water. When they get within a certain distance (about 1 cm.) they run together. If the water does not wet either body they will still attract each other; but when two bodies, one of which is wet and the other is not, are brought near together they will appear to repel each other.

These actions can be explained in the following way. Let P and Q be two plates suspended by threads near together in a liquid.

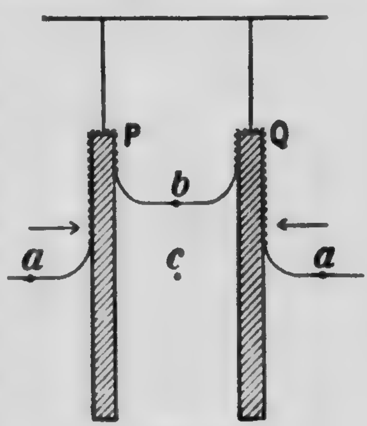


FIG. 124.

First, let the liquid wet both plates (Fig. 124). Let a, a be points on the surface of the liquid at its ordinary level, away from the plates, and c be a point on the same level in the liquid between the plates. As the liquid is in hydrostatic equilibrium the pressures exerted by the liquid at these three points must be equal, each being equal to that of the atmosphere. If one ascends from c towards b the pressure diminishes, while if one descends below c the pressure increases.

Consequently the pressure of the liquid between the plates is less than that of the atmosphere which presses on the outer surface of the plates, and the plates will be pushed together, as indicated by the arrows.

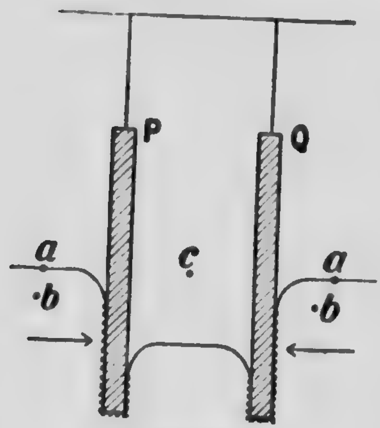


FIG. 125.

Next, take two plates which are not wet by the liquid (Fig. 125). These may be plates of glass, or aluminium, covered with paraffin. Here the pressures at a, a, as also at c between the plates, are all equal, each being that of one atmosphere. Hence the pressures at b, b in the outer liquid are greater than the pressure on the same level between the plates, and the plates will consequently be pushed together, as before.

Finally, let the liquid wet one plate but not the other (Fig. 126).

When the plates come sufficiently near together the surface of the liquid between the plates assumes the form shown in Fig. 126. It then has no level portion.

FIG. 126.

The tension of the surface on the outside pulls the plates with the force T in the horizontal plane. This tends to draw the plates apart. The tension of the

surface between the plates exerts an equal force, but in a direction making, let us say, the angle a with the horizontal. Resolving this in the horizontal plane, the force drawing the two plates together is $T \cos a$, and as this is smaller than T acting in the opposite direction, the plates will be drawn apart.

Experiment 1.

Obtain two hollow glass balls about 2 cm. in diameter and cover one with paraffin. Attach a weight to each (with wax or otherwise) so that they may float rather more than half immersed. They will appear to repel each other. If both are clean glass or both paraffined they will attract each other.

All the above results can be deduced at once from the principle that the potential energy of a system of bodies tends to a minimum.

When a clean plate is wet by a liquid in which it is held a film of the liquid spreads all over it, not just up a short distance. In Fig. 124 the entire surface of the liquid consists of the surface as ordinarily considered, and in addition a thin film extending over the plates as indicated by the dotted lines. The plate acts just as if a layer of paper or cloth covered its surface. In Figs. 125, 126 the entire surfaces are also shown. When the plates approach, the liquid rises in Fig. 124 and falls in Fig. 125 When they separate the portion between in Fig. 126 becomes horizontal. In each case there is a reduction in area, and therefore in potential energy.

7. Magnitude of Surface Tensions.

Experiment 1.

Perhaps the simplest way to obtain an approximate value of the surface tension for water is to use a bent wire of the

form shown in Fig. 127, with a wire mn laid across it. Dip it in a soap solution, and when a film adheres tilt the wire until its plane is as nearly vertical as possible. The wire will run down until the tension of the film on it just balances its weight. If it is not heavy enough a small weight may be attached to the middle of it by means of a fine thread.

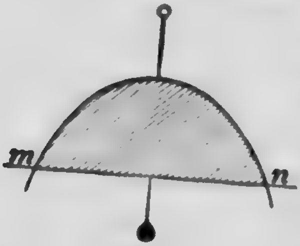


FIG. 127.

Let its weight be w grams and the length of the part between the two points of the bent wire on which it rests be l cm. Then, as there are two surfaces exerting a tension, each surface supports $\frac{1}{2}w$ grams. Hence the tension of the surface per cm. $= \frac{1}{2}\frac{w}{l}$ grams, or $= \frac{1}{2}\frac{wg}{l}$ dynes (where $g = 980$).

A more common as well as more accurate method is to observe the rise of the liquid in a capillary tube and then use the formula given above (page 244). This requires the angle of contact to be known. For water and other pure liquids it may be taken as zero.

A good method, especially for pure substances, is by observing the wave-length and the frequency of small waves or ripples on the surface of the liquid. The rate of propagation of such waves depends on surface tension. The relation between the surface tension T, the frequency n, the wave-length λ, the density ρ, and the acceleration of gravity g is

$$T = \frac{\lambda^2\rho}{2\pi}\left(n^2\lambda - \frac{g}{2\pi}\right),$$

though to obtain this formula would lead us much beyond the scope of this work. (See Encyclopedia Britannica, Art. *Capillary Action*, Vol. V, p. 273.)

The values of the surface tensions of various liquids when in contact with air, water or mercury are given in the following table :—

TABLE OF SURFACE TENSIONS AT 20° C. (In dynes per cm.)

Liquid.	Density.	Tension of Surface Separating the Liquid from		
		Air.	Water.	Mercury.
Water........ ...	1	81	..	418
Mercury.........	13.6	540	418	...
Carbon Bisulphide	1.27	32	42	372
Chloroform.	1.49	31	30	399
Alcohol..........	.79	26	..	399
Olive Oil.........	.91	37	21	335
Turpentine.......	.89	30	12	250
Petroleum........	.80	32	28	284

The values in this table are according to Quincke, a German scientist. Quincke also gives a definite angle of contact with glass for each liquid, but more recent experiments with perfectly pure liquids and clean glass lead to the conclusion that under such conditions the angle is zero.

8. Pressure within a Bubble.

As a bubble tends continually to contract it is evident that its inner surface must exert a pressure upon the air within it. Let us calculate the magnitude of this pressure in a spherical bubble in terms of the surface tension of the film.

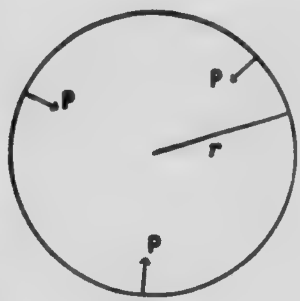


FIG. 128.

Let the radius be r cm. (Fig. 128). Then the area of each surface, outside and inside $= 4\pi r^2$ sq. cm.

Suppose, now, the sphere to contract a small amount x, where x is small compared to r. Its radius then becomes $r - x$, and the area of each surface

$$= 4\pi (r - x)^2 \text{ sq. cm.}$$

Hence the decrease in area of each surface

$$= 4\pi r^2 - 4\pi (r - x)^2,$$
$$= 4\pi (2rx - x^2),$$
$$= 4\pi x (2r - x),$$

and as x is small compared to r this approximately

$$= 8\pi rx \text{ sq. cm.}$$

The total decrease in area of both surfaces is twice this or $16\pi rx$ sq. cm.

Now the work done is equal to the decrease in the potential energy, and this (see page 241) is equal to the decrease in area × surface tension.

Hence the work done in the total reduction of area of $16\pi rx$ sq. cm. $= 16\pi rx\text{T}$ ergs.

Again, let the pressure exerted by the film upon whatever is within it be P dynes per sq. cm. Then since the area of the surface

$$= 4\pi r^2 \text{ sq. cm.,}$$

the entire force inwards $= 4\pi r^2\text{P}$ dynes.

This acts through a distance x cm., and hence does work $= 4\pi r^2\text{P}x$ ergs.

Equating the two expressions for work done,

$$4\pi r^2\text{P}x = 16\pi rx\text{T},$$

$$\text{or P} = \frac{4\text{T}}{r}.$$

Thus we see that **the pressure varies inversely as the radius of the bubble.**

In the case of a spherical drop of water there is only one surface exerting a tension and so the pressure exerted inwards upon the water within $= \frac{2T}{r}$ dynes per sq. cm.

Experiment 1.

Take a capillary tube bent in the form of a U, and fill it by drawing water through it. Then put a large drop of water on one end of the tube and a small drop on the other.

Which gains in size? Why?

In place of the bent tube two straight pieces may be joined with a piece of rubber tubing.

9. Illustrations of Surface Tension.

Experiment 1.

If small fragments of camphor are placed upon the surface of clean water they at once move about almost as if alive. The camphor dissolves slowly in the water, and the surface tension of a solution of camphor is smaller than that of pure water. Consequently if the camphor dissolves more rapidly at one side of the fragment than at the other, the surface tension on the first side will be diminished and the greater surface tension on the other side of the fragment will draw the fragment away.

This can be easily shown by rinsing a glass at the tap, filling it with water, and then scraping with a pen-knife small fragments of camphor which are allowed to fall upon the surface. They dart about, but if the surface of the water be touched with the finger the movements will likely cease, being arrested by the grease from the finger communicated to the water. Very little grease is required. Lord Rayleigh found that 0.8 milligram of olive oil on a circular surface 84 cm. in diameter was sufficient. From this he calculated that an oily film 2 millionths of a millimetre in thickness is sufficient to arrest the camphor movements.

Experiment 2.

The surface tension of alcohol is much smaller than that of water (see Table, page 249). Scatter lycopodium powder over the surface of a thin layer of water, and then place a drop of alcohol on the surface. At the place where the alcohol is, the tension is immediately reduced, equilibrium is destroyed and the superficial film of the liquid is set in motion. This will be shown by the lycopodium powder. If the water is very shallow this motion will drag the water away from the place where the alcohol is, and will lay bare the bottom of the vessel.

Experiment 3.

Rinse a glass under the tap and fill it with water, and scatter lycopodium powder as in the last experiment. Now touch the middle of the surface with a finger which has been rubbed against the hair. Enough grease will come off the finger to contaminate the water, and reduce its surface tension, and the surface layer will be drawn away from the place where the finger touched the surface. A patch will be entirely free from the powder.

Experiment 4.

Hold a drop of ether close to the surface of water. The vapour of the ether condenses on the surface, reduces the surface tension and causes an outward motion, producing a dimple on the surface.

Fig. 129.

Experiment 5.*

Pour clean water on a level board so as to form a shallow pool 2 inches wide and 2 or 3 feet long. Near its middle lay a scrap of paper and on one end place a cake of soap (Fig. 129).

The paper is soon seen to move along the surface away from the soap.

*This and the next experiment are due to Professor Kenrick.

Here the soap in dissolving weakens the surface film, and the tension in the other portion draws the surface layer away from the soap.

Experiment 6.

Cut a piece of paper into the shape of a fish (Fig. 130). On its tail put a drop of amyl alcohol (or of fusel oil) and place it on the surface of clean water. The fish swims about in a very interesting way. Why does it stop at last?

Fig. 130.

Cut the shape of an "S" from paper (Fig. 131), and put a drop of amyl alcohol on each end of it. It spins about like a pin-wheel.

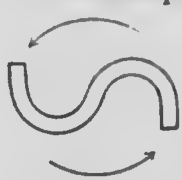


Fig. 131.

By using a shallow dish and a vertical attachment these motions can be projected on the screen.

Where the amyl alcohol is placed the surface film is weakened, and the tension in the other parts of the surface draw the surface film away from these places, causing the motion of the pieces of paper.

Experiment 7.

A is a glass bulb, with a smaller one beneath it, on the end of a small glass tube (Fig. 132). Mercury in the lower bulb makes the tube float upright in water. At *d* is a piece of wire gauze attached (by wax) to the small tube.

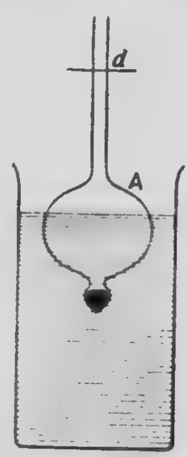


Fig. 132.

When floating in water a considerable part of the large bulb A is above the surface, and it requires quite a force to push it down.

Now press it down until the gauze touches the surface. The water wets it and clings to each

wire. This tension will be sufficient to hold the tube down in the water.

While down put a few drops of ether (or alcohol) on the surface of the water. At once the gauze breaks away and rises as shown in the figure. Adding the ether weakens the surface tension.

The size of the piece of gauze will depend on the size of the bulb and the amount of mercury holding it down; but it will be easy to find suitable dimensions.

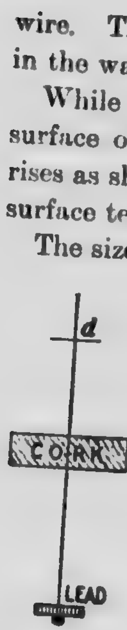


FIG. 133.

A simpler form of the apparatus is shown in Fig. 133. It consists of a hat-pin through a cork, with a piece of lead to keep it upright. In place of the wire gauze a cardboard disc *d* may be used. This can be pared down until it is just able to hold the cork down.

In this case the disc is held by the tension exerted only around its edge, while with the gauze the surface clings to each wire and so the total tension is greater.

Experiment 8.

Two simple methods of removing grease from cloth are based on surface tension. The fatty oils have a greater surface tension than benzine. Hence if one side of a grease-spot on a piece of cloth is wetted with benzine the tension is greatest on the side of the grease. Consequently the portions consisting of a mixture of grease and benzine will be drawn towards the grease and away from the benzine.

In order to cleanse the grease-spot, first apply the benzine in a ring all round the spot, and gradually bring it nearer to the centre of the spot. The grease will be chased to the middle of the spot and if a fibrous substance such as blotting-paper is placed in contact with the cloth, the grease will escape into it. If the benzine had been applied to the centre of the spot the grease would have been spread out into the cloth.

The second method is to apply a hot iron to one side of the cloth and blotting-paper to the other. The surface tension diminishes as the temperature rises. Hence the grease draws away from the hot iron and escapes into the blotting-paper.

Try these two methods.

Applications in Agricultural Processes.

10. Water in the Soil.

Surface tension undoubtedly plays an important part in agricultural processes. The moisture in a soil may be considered as attributable to three sources:—Gravitation water, capillary water, and hygroscopic water. Gravitation water is that portion in excess of the amount which the soil is able to retain under existing conditions and is consequently free to drain away. The capillary water is that part which would be retained in the small spaces between the grains of the soil, and which is capable of movement through the action of its surface tension. The hygroscopic water is that found on the surface of the grains, and which is not movable either by gravity or capillary forces.

The proportion of the moisture present at any time, which is to be credited to each of these three sources varies greatly with the circumstances, and no sharply-drawn line separates one from the other. We shall not consider the hygroscopic water any further, though it has been found that air-dried samples of soil from which all visible evidences of moisture have disappeared, will, by prolonged heating at temperatures above the boiling-point of water, be still further reduced in weight by 8 or 10 per cent.

11. Capillary Water Between Small Bodies.

Take two perfectly clean glass spheres, and put a drop of clean water on each. It spreads out, forming a very

thin film over the entire surface (Fig. 134*a*). Next, let the two spheres be brought close together. The two

FIG. 134*a*.

FIG. 134*b*.

water films unite to form a single one, and in order that the extent of the surface, and hence the potential energy, may be the least possible, the water is drawn into the space between the two spheres (Fig. 134*b*). If more

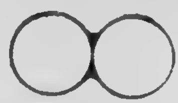


FIG. 134*c*.

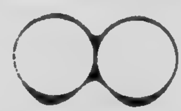


FIG. 134*d*.

water is added it takes the form shown in Fig. 134*c*. As the amount is still further increased, the force of gravity makes itself more evident, and the water collects at the lower face (Fig. 134*d*), ultimately forming drops and falling away. If a number of spheres are arranged in a layer touching each other the water will gather in the spaces where they touch, and also on the lower face, as the amount of the water is increased.

Suppose now we take two layers of such spheres (Fig. 135). The water will gather on the surfaces adjoining the points of contact, being held there by surface tension, and that portion which the surface tension cannot hold up will run down, under gravity, to the lower surface of the bottom spheres, there gathering into drops and falling away as the amount of water is increased. The amount of water

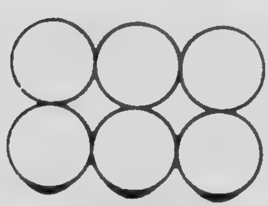


FIG. 135.

gathering on the bottom is considerable, and if the spheres are arranged in two layers they will not hold as much water as when arranged in a single layer.

12. The Action in the Soil.

We must consider the soil to be made up of small bodies with small interstices between. Suppose, now, we pack 100 cubic inches of soil into 100 cubical boxes without top or bottom, each containing one cubic inch.

Let us slowly add water to each until at last it is just ready to drip from the bottom. Then the soil is saturated.

There are air-spaces all through the soil, so that surface tension can act throughout the volume, but the greatest area of exposed surface is at the top and the bottom, and there the action of surface tension is most effective.

Now let the 100 cubic inches be built up into a vertical column 100 inches high. Instead of having 200 square inches of free surface there are only 2, and the total action of the surface tension is greatly reduced. As a consequence the soil cannot support all the water in it and it begins to drain away under the force of gravity.

13. Evaporation at the Surface.

The water at the upper surface evaporates, and its place is supplied, as far as possible, by water drawn up by surface tension. The depth from which water can be raised by capillary action differs in different soils and

for different conditions of the soil. The finer the texture is, the higher the possible rise.

Experiment has shown that capillary movement can take place through a column 5 feet in height. In this case the soil must be moist to begin with. On the other hand, if the soil is well dried the capillary rise may be less than 1 foot.

It has been shown, also, that evaporation from soil takes place entirely from the layers very near the surface.

14. Retaining the Moisture in the Soil.

The problem of preventing the rise of the water to the surface and its loss by evaporation is a very important one, especially in those countries where there is no rainfall for months in succession or where the entire yearly rainfall is small, not more than ten or fifteen inches.

It has been found that if a soil after a rain is exposed to very arid conditions, with a high surface temperature and a hot dry wind, the soil at the surface will lose water much faster than it can be brought up from below by capillary action, and a layer of dry soil may be formed on the surface which will be so dry that it will act as a protecting covering.

One of the most effective means of conserving soil moisture, however, is by "mulching," *i.e.*, by covering the surface of the soil with some loosely packed material such as straw, leaves or stable manure. The spaces between the parts of such substances are too large to admit of capillary action, and hence the water conveyed

to the surface of the soil is prevented from passing upwards any further, except by slow evaporation through the mulching layer. A loose layer of earth spread over the surface of the soil acts in the same way, and the same effect may be attained by hoeing the soil or stirring it to the depth of one or two inches with harrows or other implements.

In the semi-arid regions of the United States, Argentina, the Canadian West and other countries, in which the average rainfall lies between 10 and 20 inches, good crops of selected grain can be grown by proper cultivation.

In some cases only one crop can be grown in alternate years, the year of no crop being used to preserve the moisture in the soil. In our Canadian West during a dry season it is found that land which was "summer-fallowed" the year before produces the heaviest crop.

Application to Dyeing and Filtration.

15. The Process of Dyeing.

There is great variety both in the materials to be dyed and in the colouring matter to be applied to them, and we are not surprised to find that the phenomena observed in the process of dyeing are very complicated. No single hypothesis as to the nature of the action taking place will account for all the results obtained.

In some cases chemical action undoubtedly takes place; in others the process is probably physical, and there is evidence that capillary action or surface tension is of great importance.

Experiment 1.[1]

Into vessel A (Fig. 136), pour clean water, and into vessel B a weak solution of saponine (1 gram of saponine to 500 c.c. of water.)

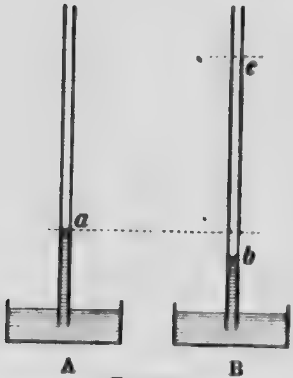


Fig. 136.

Hold a capillary tube in A. The water rises to level *a*. Then remove the tube and hold it in B. The liquid now rises only to level *b*, considerably below level *a*.

This shows that the saponine solution has a smaller surface tension than clean water has.

Now draw the solution in B up to the level *c* and let it go suddenly. The column rapidly falls to level *a* and then settles less rapidly down to *b*.

While falling from *c* to *a* the liquid at the surface is being renewed constantly, and so the constitution of the surface layer is very approximately the same as that of the solution generally, which is little different from pure water. However, in a few seconds some of the saponine concentrates at the surface and produces a reduction in the surface tension. This gradual reduction is seen in the slow sinking of the column to its final height.

From this experiment we get a very important result. **When a substance, on being dissolved in water, reduces its surface tension, there is a concentration of the substance in the surface layer.**

This conclusion, indeed, is predicted from theoretical considerations based on the laws of thermo-dynamics, and it can be verified by many other experiments. As stated above (page 236) a system always endeavours to change so as to have the least possible potential energy. When such a substance

[1] The experiments in this section were suggested by Prof. Kenrick.

goes into the surface it reduces the surface energy, thus contributing to a reduction in the total potential energy.

It is to be observed that the surface layer is excessively thin so that the actual amount of matter concentrated there need not be great.

Experiment 2.

Moisten both sides of a piece of paper and lay it on the surface of clean water in a large photographic tray. Allow a thread, attached to one side of it, to hang over a glass rod or over the edge of the tray if it is very smooth.

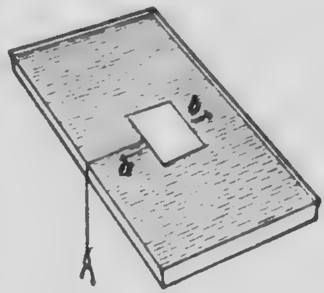


FIG. 137.

The tension of the surface is the same in all directions in its plane, and hence is the same in the two directions a and b (Fig. 137). The slightest force on the paper will move it in the direction in which the force acts.

Add a small weight (a bit of bent wire) to the end of the thread. If this is sufficient to overcome the friction of the thread on the glass rod, the paper will move in the direction a with a certain speed.

Next, in place of the pure water use a solution of methyl violet (1 gram to 4 litres), which reduces the surface tension of the water. Allow the solution to stand for a few minutes before placing the paper on it.

Observe the rate at which the paper is drawn aside. It is not so great as before!

This arises in the following way. When the paper is displaced in the direction a it exposes new surface at b. This at first is practically the same as pure water, which has a greater surface tension. Hence the surface tension pulling the paper in direction b is greater than that in direction a, and the

motion takes place only as the newly exposed surface becomes concentrated and so is the same as on the other side of the paper.

This explanation can also be verified by removing the weight and then pulling the thread by the hand. On letting go the water-surface-tension at b will draw back the paper in that direction.

Place a match on the surface and drive it endways. It sticks as though there were a scum on the surface.

Finally stir the solution and try the experiment with the paper and the match at once, *i.e.*, before the surface has become concentrated. It acts like that of pure water.

Experiment 3.

Make a solution of methyl violet (1 gram to 4 litres of water). About one-third fill a large separ ing funnel (Fig. 138). Shake vigorously, causing froth to gather above the liquid.

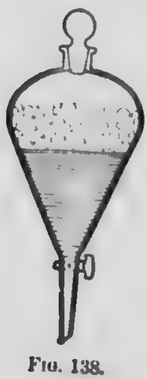


FIG. 138.

Let it stand 4 or 5 minutes to allow the liquid between the bubbles to run down. Then drain off all the liquid which has collected. Call this solution A.

Next, let it stand for 4 or 5 minutes more, to allow the froth to settle, and then draw off the liquid formed from it. Call this solution B.

Now make a solution of 1 c.c. of A to 20 c.c. of water, and pour in one side of a double glass vessel with plane sides (Fig. 139).

Make a solution of 1 c.c. of B to 20 c.c. of water, and pour in the other side of the vessel.

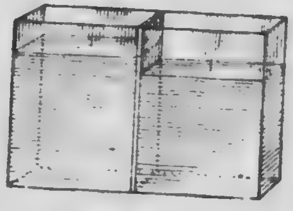


FIG. 139.

Place the vessel in the lantern, and project on the screen, or hold it in front of a piece of white paper or up to the window.

It will be found that the second solution is of a deeper colour than the first.

This result is explained as follows:—Methyl violet when dissolved in water reduces the surface tension of the water, and any substance which does that concentrates at the surface. The bubbles of froth have much surface compared to their mass, and the methyl violet is concentrated on their surfaces.

Hence the liquid formed from the bubbles contains more of the dye per c.c. than does the liquid first drained off.

The proportions in the two solutions compared must be accurately the same as the difference in colour is only slight. Use a 1 c.c. and a 20 c.c. pipette, previously rinsing out with some of the liquid to be measured.

If froth does not form on the solution make a new one with fresh water.

Experiment 4.

Place a drop of a weak solution of red ink on white filter or blotting paper, and observe how it spreads. When the action has ceased it will be found that the red colouring matter has spread a certain distance, but the water in the solution has gone a considerable distance farther.

Again, place drops of a dilute solution of barium hydroxide and an alcoholic solution of phenol phthalein near together on filter paper and allow them to spread into each other. The beautiful pink colour resulting when these two substances combine does not appear at the edge of the drop of barium hydroxide, but some distance within it. The outer portion of the round spot coming from the barium hydroxide is pure water, the solid having been left behind, nearer the centre of the spot.

Many solutions of salts or of dyes exhibit this phenomenon, the water diffusing amongst the fibres of the paper and leaving the dissolved substance behind upon the fibres.

Still more striking and beautiful effects are obtained with solutions of two dyes. Make a dilute solution of picric acid

and crocein scarlet, and put several drops on white filter paper (supported on the top of a beaker). When the spreading has ceased there will be seen a large spot of red with a yellow fringe, and this surrounded by clear water. The picric acid diffuses more freely than the scarlet.

A solution of acid magenta and indigo sulphate of soda will give an indigo spot fringed with magenta.

Instead of putting drops on the paper, a strip of filter paper may be suspended with its lower end in the solution. The clear liquid rises highest and usually one colour higher than the other, if two are present.

These experiments are easy to perform, and the results are beautiful and suggestive.

The action illustrated in the above experiments is almost certainly present in some cases of dyeing. The coloured solution comes in contact with the surface of the material to be dyed; the tension of the surface there is reduced and the colouring matter concentrates at the surface and is deposited on the material. Probably, with some materials the water of the coloured solution passes freely through the capillary spaces leaving the particles of the dye behind on the material.

16. Surface Tension in Filtration.

Filters can be divided into two classes.

In filtering solid impurities, or a precipitate, from a liquid, the filtering material (paper, cloth, sand, etc.) has interstices through which the liquid can pass but the solid particles cannot. Surface tension does not enter here.

It has been known for many years that neutral filters, such as sand in layers, will remove colouring matter and

to some extent salts in solution. This filtering action is undoubtedly intimately connected with the large amount of surface of the particles presented to the liquid, the greater the surface the stronger being the action.

If a dilute solution of acetic acid is filtered through fine white sand, nothing but pure water will percolate through, the whole of the acid being kept back by this action. Dilute solutions of various other substances show a similar action.

In experiment 4, above, the red matter in the ink, and the solid in the solution of barium hydroxide were held back while the pure water flowed on.

The action in these cases is certainly a surface phenomenon, probably explainable in the same manner as the phenomena of dyeing just described above.

It may be well to remark, however, that in the purification of water by filtration other considerations enter. For a long time this was looked upon as a mechanical process of straining out the solid particles and thus rendering turbid water clear. But now it has been shown that in sand-filtration of water on a large scale an essential feature is the presence in the upper surface layer of the sand of colonies of bacteria forming jelly-like masses. Not until a fine film of mud and microbes has been formed upon the surface of the sand are the best results obtained.

EXERCISE XLII.

1. Explain why the end of a stick of sealing-wax when held in a flame becomes rounded.

2. Why are small drops of mercury resting on a horizontal surface more nearly spherical than larger ones?

3. Calculate the work done in blowing a soap bubble 10 cm. in diameter.

4. Two drops of mercury 1 mm. and 2 mm. in diameter, respectively, coalesce. Compare the pressure within the liquid due to surface tension in the two original drops and in the one formed by their union.

5. When a soap bubble bursts the water from it is thrown in every direction. Account for this.

6. Calculate the heights to which pure water, alcohol and turpentine will rise in capillary glass tubes 1 mm. in diameter. (For surface tensions and densities, see Table on page 249.)

7. One soap bubble, 8 cm. in diameter, is on one end of a U tube, and another, 3 cm. in diameter, is on the other end. If there is a free passage from one to the other, which one will increase in size?

8. Two parallel plates, separated by a space d, stand vertically in a liquid, having density ρ, surface tension T and angle of contact θ. Show that the height h to which the liquid will rise is

$$h = \frac{2T \cos \theta}{g\rho d}.$$

Compare this with the height in a cylindrical tube whose diameter is equal to the distance between the plates.

(Consider the equilibrium of a portion of the liquid between the plates 1 cm. in length.)

Solution for Soap-Films and Bubbles.

A solution of Castile soap and rainwater, with some Price's glycerine added to make the film last longer, will probably answer all purposes; but for the very best results a specially prepared solution is desirable.

The following is the recipe recommended by Reinold and Rücker and by Boys. Fill a stoppered bottle three-fourths full with distilled water, add one-fortieth by weight of fresh oleate of soda, and leave for a day to dissolve. Nearly fill the bottle with Price's glycerine, and shake well. Leave the bottle a week in a dark place, and then with a siphon draw off the clear liquid from under the scum into a clean bottle, add a drop or two of strong ammonia solution to each pint, and keep carefully in the stoppered bottle in a dark place, filling a small working bottle from it when required, but keeping the stock bottle undisturbed and never putting any back into it. Do not warm or filter the solution and never leave the stopper out or expose the liquid to the air.

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Encyclopedia Britannica, 11th Ed. Art. "Capillary Action," Vol. V, p. 256.

C. V. Boys, *Soap Bubbles*. (Full of fine experiments with instructions for performing them.)

Edwin Edser, *General Physics for Students*, Chapters IX and X.

Maxwell, *Theory of Heat*, Chapter XX.

Tait, *Properties of Matter*, Chapter XII.

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W. P. Dreaper, *Chemistry and Physics of Dyeing*.

CHAPTER XIX.

THE FLOW OF FLUIDS.

1. Services Obtained from Flowing Fluids.

From an economic point of view the study of the laws of flowing fluids is of great importance. Immense stores of energy are present in the waters of our rapid rivers, and in order to utilize it we must know the laws according to which they move. In the systems of waterworks in our cities and towns the water is pumped into iron pipes, from which it is drawn for domestic use, for running elevators and water motors, and for other industrial purposes.

Air and steam, forced through pipes, are used for actuating drills, for driving turbine and ordinary engines, for applying the brakes on railway trains and street cars, for heating buildings and for numerous other purposes.

Again, our winds are currents in the air, their motion being shown in the swaying of trees, and in the sweeping onward of clouds in the sky or of clouds of dust and smoke at the surface of the earth.

It is therefore evident that a knowledge of the laws in accordance with which fluids move is of the highest value. The phenomena, however, are very complicated, and the determination of their laws is a matter of difficulty.

2. Steady Motion.

Consider the water moving forward in a river or flowing in a pipe which has a varying diameter, and which, perhaps, has a varying direction. If we could colour the particles of water in successive cross-sections, thus rendering it possible to trace their motions, we would probably be surprised to see the extraordinary way in which some of them eddy about instead of simply moving forward. The particles near the shore and bottom of the river, or near the surface of the containing pipe, are continually being thrown into eddies.

But it is evident that if the source of supply is perfectly constant, the flow will be continuously uniform, and the particles in one cross-section will follow approximately the same paths as those in the preceding one. For example, if a vessel is kept constantly full by allowing water to run uniformly into it from a reservoir, and if the water is permitted to escape from an opening anywhere in the vessel, the motion of the particles which pass any fixed point in the vessel will be the same at all times. If a water-sprite could stand in the liquid and mark each particle as it came along in a certain direction to that point, all of these particles would be seen to follow the same curved path.

Motion such as that just described is called **Steady Motion** or **Simple Flow**; **and the lines imagined to be drawn in the liquid so as to be at each point in the direction of the flow, or, in other words, the lines along which the particles travel, are called Stream Lines.**

Thus, consider steady motion through a pipe with a

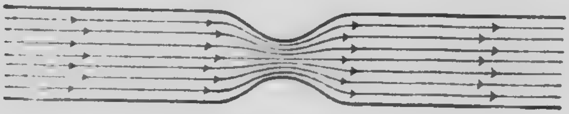

FIG. 140.

contraction, or **throat,** in it (Fig. 140). The fine lines indicate the form of the stream lines.

Let us consider the stream lines drawn through a closed curve *a* (Fig. 141) in the liquid. A particle of the fluid which commences to move along one of these lines will continue to do so. It is evident that these lines of flow taken together form a tube; it is called a **Tube of Flow.**

FIG. 141.

Since the line of flow, or stream line, passing through a point indicates the direction of flow at that point it is evident that two lines of flow cannot cross each other. If they did the resultant motion at the point of intersection would have two directions, but in steady motion the movement of the particles at a point are continually in a single definite direction.

Such being the case, there can be no flow across the bounding walls of the tube, and the particles which are within the tube at one time will continue within it. The particles composing a cross-sectional layer will continue to be a cross-sectional layer.

3. Height to Which a Jet will Rise.

Experiment 1.

Arrange apparatus as in Fig. 142. The water escapes from a small orifice. The jet rises nearly to the level of the free

surface of the liquid in the vessel, and we suspect at once that if there were no losses through friction the jet would rise exactly to that level.

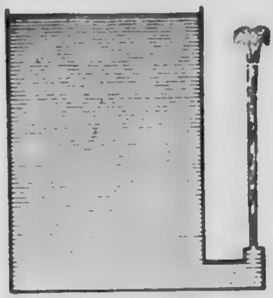

Fig. 142.

Now if a body falls through a height h it attains a velocity v where $v = \sqrt{2gh}$. In the same way, if the body is thrown upward, and rises to a height h the initial velocity $= \sqrt{2gh}$.

In the case of the jet of liquid, if h is the depth of the orifice below the free surface in the vessel, the velocity of efflux $= \sqrt{2gh}$.

This relation is rigidly true only for a perfect liquid, or one which flows without friction.

4. Flow of Liquid from an Opening in a Vessel.

The result obtained in the last section can be deduced from the principle of energy.

Let the opening be at a distance h cm. below the surface of the liquid (Fig. 143). Let the density of the liquid be ρ, the area of the free surface be A sq. cm., and the velocity of the out-flowing liquid be v cm. per sec., that is, a small speck of dust in the liquid at the opening would be carried forward at this rate.

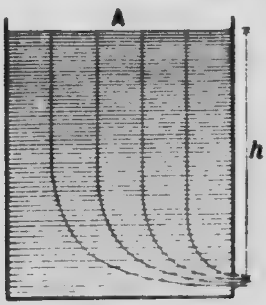

Fig. 143.

Suppose that in a very short time t the level of the free surface falls a very small distance x cm. Then the volume of liquid which has escaped is Ax c.c. Its mass is ρAx grams,

and as its velocity is v cm. per sec., its kinetic energy $= \frac{1}{2}\rho A x v^2$ ergs. (See page 57.)

This kinetic energy must be gained at the expense of the potential energy of the liquid. Now each layer has fallen through a height x cm., or the entire volume has fallen through this distance. The mass is $hA\rho$ grams, and its weight is $hA\rho g$ dynes. Hence the

$$\text{Decrease in potential energy} = ha\rho gx \text{ ergs.}$$

Therefore, $$\tfrac{1}{2}\rho A x v^2 = hA\rho g x,$$

$$\text{or } v^2 = 2gh \text{ and } v = \sqrt{2gh};$$

that is, **the velocity is the same as that which would be acquired by falling through the distance of the opening below the free surface.**

This result can be obtained in another way. In Fig. 143 are shown some of the stream lines. They all begin at the free surface and pass through the opening.

Now consider the stream lines drawn through a series of particles on the free surface of the liquid forming a closed curve. As explained above, the hollow surface formed by all these stream lines is a tube of flow, and the liquid forming the surface layer will move onward in this tube, always forming a cross-section. Let the mass of the surface layer be m grams. It begins at the surface with zero velocity, and the velocity v which it has on emerging from the opening is due to its descent through the distance h cm. The decrease in its potential energy $= mgh$ ergs; the kinetic energy on flowing out $= \frac{1}{2} mv^2$ ergs.

Hence $\frac{1}{2} mv^2 = mgh$,

$$\text{or } v = \sqrt{2gh} \text{ cm. per sec., as before.}$$

This is known as *Torricelli's Law*. It was formulated by him in 1643, 200 years before the principle of the conservati n of energy had been established.

This result was obtained on the assumption that the liquid was perfect, that there was no friction in the passage of one layer over another, or in other words, that it had no *viscosity*. As a matter of fact, water, ether, alcohol, mercury and such liquids possess very little viscosity, and the law is very nearly fulfilled by them. In the case of water the velocity is not quite that given by theory, a small amount of the energy being transformed into heat. The velocity is approximately $\frac{97}{100} \times \sqrt{2gh}$.

5. The Contracted Vein.

The rate at which liquid is escaping, however, cannot be found from knowing the area of the opening and the velocity v of the efflux. Just outside the opening the jet contracts somewhat, and we must take the area of a cross-section where it is least. The size and shape of the cross-section is modified by the shape of the opening, and the area in general can be determined only by experiment. When the opening is sharp-edged round orifice in a plane surface the area of the jet is on the average $\frac{64}{100}$ of that of the opening, or the cross-section of the jet is about $\frac{5}{8}$ of the area of the opening.

Experiment 1.

Test the rate of flow from orifices of different shapes, circular, square, triangular. This can conveniently be done by making an opening of some size (say $1\frac{1}{2}$ in. in diameter) near the bottom of a tank, and then placing over this plates with orifices of different shapes in them. The experiment in each

case should continue only a short time so that the flow may be nearly uniform. The rate of flow can be determined by taking the time and observing the fall of the water in the tank, or better, by catching the water and measuring it.

Also compare the flow through a circular orifice in a thin plate with that through a short tubular orifice of the same internal diameter.

6. Energy of a Liquid Under Pressure.

A liquid possesses potential energy by virtue of its being submitted to pressure, and the amount of this energy can be calculated in the following way.

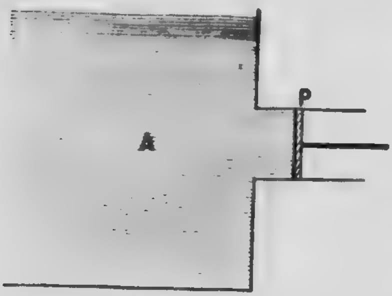

FIG. 144.

Let A (Fig. 144) be a tank in which is water under a pressure of p grams, or pg dynes, per sq. cm., and let P be the piston of a pump by which water is forced into the tank. Let a sq. cm. be the area of the piston. The total pressure on the piston is ap grams or apg dynes, and when it moves inwards through a distance x cm., it does apx gm.-cm., or $apgx$ ergs, of work.

In doing so it forces ax c.c. of water into the tank, which must possess as potential energy the energy expended in placing it where it is.

Hence ax c.c. have apx gm.-cm., or $apgx$ ergs, of P.E., and 1 c.c. has p gm.-cm., or pg ergs, of P.E.; *i.e.*, **the measure of the potential energy per unit of volume possessed by a liquid is the same as the measure of the pressure per unit of area to which it is subjected.**

Thus, if a liquid is under a pressure of 10,000 dynes per sq. cm., each c.c. of it possesses 10,000 ergs of potential energy. If the pressure is 60 pounds per sq. ft., each cu. ft. possesses 60 ft.-pds. of potential energy.

Examples of this effect are often seen. When water from the city waterworks at a pressure of, say, 100 pounds per sq. inch, is admitted to the cylinder of an elevator in a high building, it performs work in raising the car of the elevator to the upper stories of the building. Or, when pumped into the cylinder of a hydrostatic press, immense pressures are produced, which are used in compressing bales, etc.

7. Energy of a Liquid in Motion.

Let the velocity be v cm. per sec., and m grams be the mass of 1 c.c. (*i.e.*, the density) of the liquid.

Then the kinetic energy per c.c. $= \frac{1}{2}mv^2$ ergs.

If the velocity is v ft. per sec. and the density is ρ lbs. per cu. ft.

$$\text{Then the kinetic energy per cu. ft.} = \tfrac{1}{2}\rho v^2 \text{ ft.-poundals,}$$

$$= \tfrac{1}{2}\frac{\rho v^2}{g} \text{ ft.-pounds,}$$

since 1 pound force $= g$ poundals.

8. Rate of Flow of a Liquid.

First, consider steady flow in a tunnel or a pipe of uniform cross-section (Fig. 145). Let the area be a sq. cm., and the velocity be v cm. per sec.

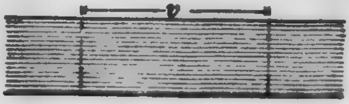

FIG. 145.

Then the amount which flows past any point in 1 sec. is av c.c. In practical engineering work the rate of flow is usually stated in cu. ft. or cu. metres per sec.

Next, let the pipe have a contracted portion or throat, as in Fig. 146.

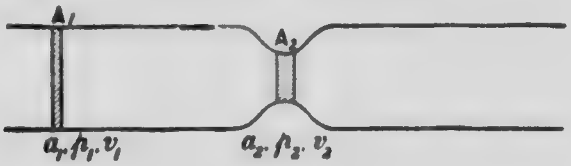

FIG. 146.

Let the area of the cross-section at A_1 be a_1 sq. cm., the velocity there be v_1 cm. per sec., and the pressure there p_1 dynes per sq. cm.

At A_2 let the corresponding values of these quantities be a_2, v_2, p_2.

Now the same quantity flows past A_1 and A_2 during 1 sec. Hence $a_1v_1 = a_2v_2$.

But a_1 is greater than a_2; hence v_2 is greater than v_1, and we have the law: **The velocity of the liquid is inversely proportional to the area of the cross-section.**

9. Relation Between Pressure and Velocity.

Suppose the tube in which the liquid is flowing is horizontal, and consider the motion of 1 c.c. of the liquid along the axis, from the centre of the section at A_1 to the centre of that at A_2.

Its energy at $A_1 = p_1 + \frac{1}{2}\rho v_1^2$ ergs......(1)

Its energy at $A_2 = p_2 + \frac{1}{2}\rho v_2^2$ ergs......(2)

But these must be equal, and therefore

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2 \quad\text{........(3)}$$

= the corresponding expression for any section.

Hence the quantity

$p + \frac{1}{2}\rho v^2$ is a constant for any section ......(4).

The relation (3) can be written

$$p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) \ldots\ldots (5).$$

But since the area at A_2 is smaller than that at A_1, the velocity v_2 is greater than the velocity v_1, and also v_2^2 is greater than v_1^2. Hence p_1 is greater than p_2, and we obtain the law that **when the velocity increases the pressure diminishes.**

The pressure exerted by the liquid at a contracted portion of the pipe is less than where the pipe is larger. This is entirely contrary to the view commonly held. Most people think that when the liquid enters a contracted portion its particles are squeezed together and it exerts a greater pressure against the walls of the pipe. This view, however, is quite erroneous.

If we use the English units, taking p in pounds'-weight per square foot, ρ as pounds of mass per cu. ft., and v in feet per sec., then

$$\begin{aligned} \text{P.E. of 1 cu. ft.} &= p \text{ ft.-pounds,} \\ \text{and K.E. of 1 cu. ft.} &= \tfrac{1}{2}\rho v^2 \text{ ft.-poundals,} \\ &= \tfrac{1}{2}\frac{\rho}{g}v^2 \text{ ft.-pounds.} \end{aligned}$$

Then the energy possessed by 1 cu. ft. of the liquid

$$= p + \tfrac{1}{2}\frac{\rho}{g}v^2 \text{ ft.-pounds of energy,}$$

and the relation (3) becomes

$$p_1 - p_2 = \tfrac{1}{2}\frac{\rho}{g}(v_2^2 - v_1^2) \ldots\ldots\ldots (6).$$

In the above formula (3) we have not taken into account the force of gravity. Suppose at A the unit of

volume of the liquid is at a height h above a horizontal plane of reference taken as a standard level, then its P.E. due to gravity is, in the C.G.S. system $g\rho h$ ergs, or in the English units ρh ft.-pounds. The entire energy possessed by the unit of volume, therefore,

$$= p + g\rho h + \tfrac{1}{2}\rho v^2 \text{ ergs}, \ldots\ldots (7)$$

$$\text{or} \quad = p + \rho h + \tfrac{1}{2}\frac{\rho}{g} v^2 \text{ ft.-pounds}, \ldots (8)$$

in the respective systems of units.

The relation between pressure and velocity, expressed by the equation,

$$p + g\rho h + \tfrac{1}{2}\rho v^2 = \text{constant},$$

is known as **Bernoulli's Theorem.** It was demonstrated in 1738 by Daniel Bernoulli, and lies at the basis of the science of hydraulics.

10. Application of Newton's Second Law.

That the pressure exerted by a liquid diminishes as its velocity increases can be deduced directly from Newton's Second Law of Motion.

As the liquid passes from A to B (Fig. 147) its velocity increases, or it has an acceleration and its momentum is continually increasing. Hence an unbalanced force acts in the direction from A to B. This force is the resultant pressure to which the portion of the liquid under consideration is subjected by the surrounding liquid, and if the pressure at B were as great as at A the motion would not take place.

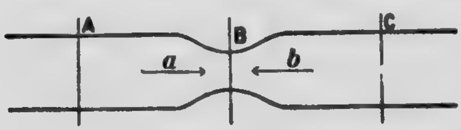

Fig. 147.

Again, on passing from B to C the velocity decreases; hence there must be an unbalanced force acting in the backward direction, as shown by the arrow b, and so the pressure at C, where the velocity is less, is greater than at B.

11. Deduction of Torricelli's Law from Bernoulli's Theorem.

Consider the energy possessed by unit volume of the liquid at the surface A (Fig. 143), and also when it flows from the orifice. Notice that the pressures exerted by the liquid, or to which it is subjected, at the surface A and at the orifice are the same, each being simply the atmospheric pressure, which acts throughout the system. As we propose to equate the energy at one point to that at the other, the potential energy, p, due to the pressure, may be omitted, or taken as zero.

At A, we have $p = 0$; the potential energy due to gravity or weight $= g\rho h$, taking as zero level that of the orifice; the kinetic energy, $\frac{1}{2}\rho v^2 = 0$, since the velocity of the surface is very small and v may be put $= 0$.

Hence the entire energy, given by the expression (6) becomes $g\rho h$.

At the orifice, $p = 0$; $h = 0$ and the potential energy due to gravity, $g\rho h = 0$; the kinetic energy $= \frac{1}{2}\rho v^2$.

Hence the entire energy $= \frac{1}{2}\rho v^2$.

Equating the energy at A to that at the orifice,

$$\tfrac{1}{2}\rho v^2 = g\rho h, \text{ or } v^2 = \sqrt{2gh}, \text{ as before.}$$

12. Experimental Illustrations of Bernoulli's Theorem.

Experiment 1.

Obtain a glass tube, blown as illustrated in Fig. 148, having

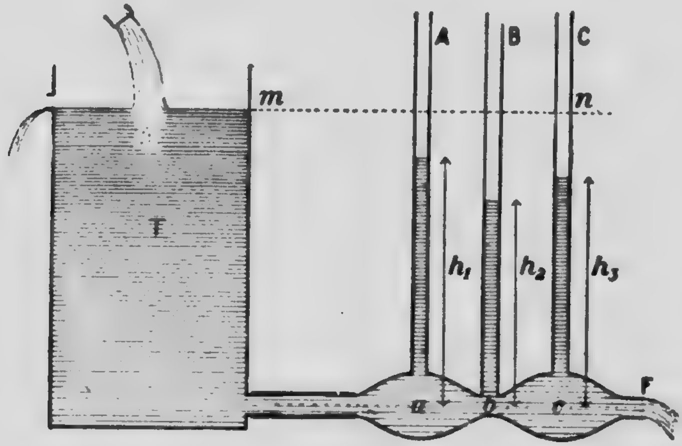

FIG. 148.

two larger portions separated by a smaller neck, with a small tube rising from each of these portions. The large portions should have a diameter as great as possible, and their lengths should be several times as great as their diameters. If the tube is too small, friction considerably affects the flow, and if the expanded portions of the tube are too "bunty" the water is thrown into eddies and the flow is far from being steady.

Connect to a tank T which is kept full of water, or attach directly to a water tap.

First, hold a finger over the end F. There will be no flow, and the water in the tubes A, B, C will rise to the line *mn*, assuming the same level as in T.

Next, let the water run freely. Now if the particles of water are crowded together as the sections of the cone get smaller and are thus subjected to increased pressure, this

would be shown in the water level in the tubes. We might expect that in B to be highest and that in A or C lowest; but such is not at all the case. They assume the levels shown in the figure. They are slightly lower than they would be if the water moved entirely without friction.

Observe that the pressures at a, b, c, etc. are those due to a 'head' h_1, h_2, h_3, etc. (cm.) respectively. Then if the corresponding pressures are p_1, p_2, p_3, etc. (dynes per sq. cm.), we have the relations

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2 = \text{(similarly for each section)} = \text{constant}.$$

In place of the glass tubes shown in Fig. 148, an apparatus such as illustrated in Fig. 149 may be used. This consists of two zinc or tin cones soldered together, 3 inches in diameter at the common base and tapering to ½ inch at the ends. The shorter is 3 inches, and the other 12 inches long. Three openings are in the longer cone. In these can be inserted corks through which glass tubes pass.

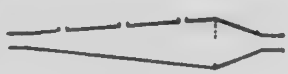

FIG. 149.

A third convenient form of the apparatus is shown in Fig. 150. It is made of glass. One end of a U tube is fused into the wide portion and the other end into the narrow portion of the tube. On causing the water to flow through, it rises in both arms of the U tube, but higher in that portion joined to the wide part of the tube. It will be observed that the pressure of the air within the U tube, exerted upon the surface of the water in the two arms,

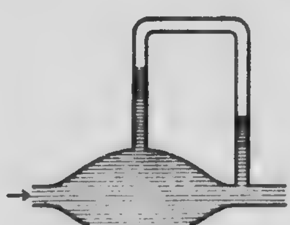

FIG. 150.

is greater than an atmosphere, but it is the same in both arms.

Experiment 2.

Another interesting experiment is illustrated in Fig. 151. A and B are two tanks, about 1 sq. ft. in horizontal section, provided with converging pipes as shown. They must be carefully placed so that the contracted openings are exactly opposite each other.

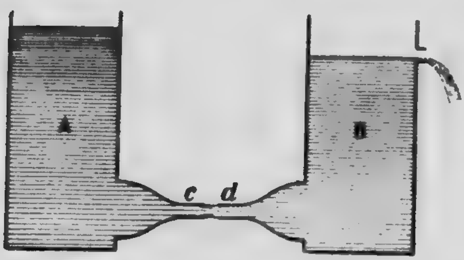

Fig. 151.

As water is supplied to A it spurts out of c into d, and continues to do so until the level in B almost reaches that in A. If the water were a perfect liquid the levels would be the same. In an actual experiment a level of 18 inches in B was maintained by a level of $20\frac{1}{2}$ inches in A, the $2\frac{1}{2}$ inches being lost by friction.

Notice that there is no waste in the water as it shoots across from c to d, except the small sprinkling caused by inexactness of aim and by want of exact circularity in the orifices. Also, in the space between c and d there is no pressure except the atmospheric pressure which acts uniformly throughout the system.

13. Examples of the Flow of a Gas.

The laws, according to which a compressible fluid, such as a gas, flows are much more complicated; but when the variations in the pressure are not too great the relation between the pressure and the velocity still holds.

Experiment 1.

Examine a Bunsen burner. The gas escapes from a small hole at the base of the burner with a high velocity. The pressure, consequently, is very low, and air rushes in through the opening in the tube, and the mixture of gas and air burns with a non-luminuous flame at the top of the tube.

Experiment 2.

In Fig. 152 a tube is fixed in a flat disc, the end of the tube being flush with the surface of the disc. A light disc of metal or cardboard is held near it by means of three metal pins which move freely through the lower disc. If a vigorous current of air is blown through the tube when it is held vertically, the lower disc will rise up to the other one. In this case the air spreads out in the space between the discs radially from the tube. As it spreads out its velocity is diminished and the pressure increased. Now at the rim the pressure is approximately that of the atmosphere, and so at the centre it must be less than one atmosphere. Hence the atmospheric pressure on the lower side pushes the disc upward.

FIG. 152.

A very simple form of the apparatus is shown in Fig. 153. A glass tube is pushed through a cork until its end is flush with the lower side. A thin layer of cork, with a pin through it to prevent it moving aside, will be drawn up to the thicker cork when a current of air is blown through the tube.

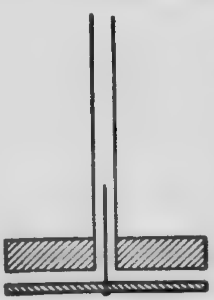

FIG. 153.

The above effect was first observed in some iron works in France, about 1826. One of the forge-bellows opened in a flat wall,

and it was found that a board presented to the blast was sucked up against the wall.

Experiment 3.

The simplest way to exhibit the effect, however, is due to Faraday. By means of the palm of the left hand hold snugly up against the palm of the right hand a piece of tissue paper 3 or 4 inches square, and then blow through the opening between the first and second fingers against the middle of the paper. Instead of being blown away, the paper will be sucked up to the hand. After a few trials the experiment will be easily performed.

Experiment 4.

Another simple experiment is shown in Fig. 154, T is a short, wide glass tube. Through a cork in one end is a glass tube A drawn out to a small opening *a*. Through a cork in the other end a wider tube B is inserted. At the bottom is a manometer C filled with coloured water. On blowing through A the liquid in C rises. Explain this.

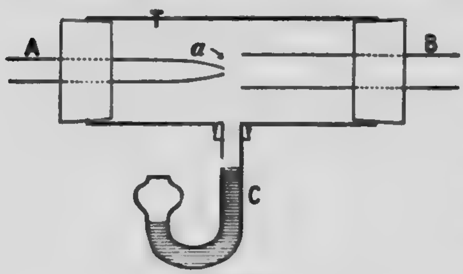

FIG. 154.

Applications of Bernoulli's Theorem.

14. The Venturi Water Meter.

The object of this instrument is to measure the rate of flow in a water-main. Its construction is shown in

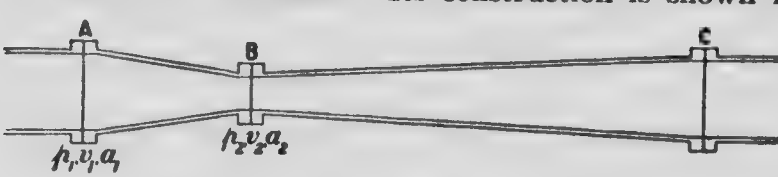

FIG. 155.

Fig. 155. Between points A and C a *throat* is inserted,

the change in the area of the pipe being gradual in order to avoid eddies. The areas of the cross-sections at A and B are carefully measured and pressure gauges are inserted at these points. Now if we know the areas of these cross-sections and the difference between the pressures there we can determine the flow in the pipe.

A numerical example will best illustrate the use of the instrument. Let the diameters of the sections at A and B be 30 and 10 cm., respectively, and the difference between the pressures be 352,800 dynes per sq. cm. To find the rate of flow.

Let p_1, v_1, a_1 be the pressure, velocity and area, respectively, at A; and p_2, v_2, a_2 be the corresponding values at B.

The diameter at A = 3 × the diameter at B

$$\text{Hence } a_1 = 9\, a_2.$$

Now $a_1v_1 = a_2v_2$, or $v_2 = \dfrac{a_1}{a_2} v_1 = 9\, v_1$.

From Bernoulli's Theorem we have

$$p_1 - p_2 = \tfrac{1}{2}\rho\,(v_2^2 - v_1^2),$$

$$= \tfrac{1}{2}\rho\, v_1^2 \left[\left(\frac{a_1}{a_2}\right)^2 - 1\right] = 40\,\rho\, v_1^2 \text{ in this case.}$$

Also, $p_1 - p_2 = 352{,}800$, and $\rho = 1$,

$$\text{Hence } 352{,}800 = 40\, v_1^2,$$

$$\text{and } v_1 = 93.91 \text{ cm. per sec}$$

Now the area $a_1 = 707$ sq. cm.;

Hence rate of flow = 707 × 93.91 = 66,394 c.c. per sec.

Notice that 352,800 dynes = 360 grams-weight ($g = 980$), and the difference in pressure is equal to that of a column

of water 360 cm. high. It is said to be that of a 'head' of 360 cm. or 3.60 metres.

Next, let us solve the question using feet and pounds, which are the units generally employed by engineers in practical work.

Take the diameters to be 12 and 4 inches and the difference of pressure to be that of a head of 12 feet.

A head of 12 feet = a pressure of $12 \times 62\frac{1}{2}$ pounds per sq. ft. The formula, with these units, is

$$p_1 - p_2 = \frac{1}{2}\frac{\rho}{g}(v_2^2 - v_1^2). \quad \text{[Equation (6) above]}$$

And $p_1 - p_2 = 12 \times 62\frac{1}{2}$, $\rho = 62\frac{1}{2}$, $g = 32$, $a_1/a_2 = 9$.

$$\text{Hence } 12 \times 62\tfrac{1}{2} = \tfrac{1}{2} \times \frac{62\frac{1}{2}}{32} \times v_1^2 \times 80,$$

$$\text{or } v_1^2 = 9.6,$$

$$\text{and } v_1 = 3.098 \text{ ft. per second.}$$

Now the area $a_1 = 0.7854$ sq. ft.

and the rate of flow $= a_1 v_1 = 2.433$ cu. ft. per second.

Observe that if the pressure be taken as due to a certain 'head' of the liquid the density cancels out of the equation.

This meter was invented in 1887 by Clemens Herschel, an American engineer, who named it after Venturi, an Italian, who first described an experiment illustrating the principle involved in it in 1797. There are other forms of water meters, but this one is especially convenient in the case of very large water mains.

15. The Jet Pump.

The principle of the Jet Pump is illustrated in Fig. 156.

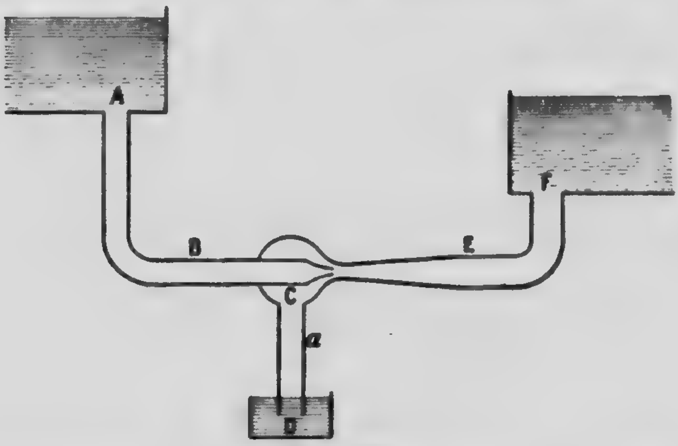

FIG. 156.

Water is led from a reservoir A by a pipe B which tapers at C. The velocity here is great and the pressure is reduced until below that of the atmosphere, which, acting upon the surface of the water in D, forces it up the pipe *a*. It mixes with the water flowing from C, and the combined stream flows on by the tube E to the reservoir F, which, however, cannot be higher than A. Thus the water is pumped from D up to F.

A simple apparatus for showing the action of this pump is illustrated in Fig. 157. The tube B is attached to a water tap (or a supply of compressed air), and the tube A is placed in the liquid to be pumped. To start the pump it may be necessary to fill it with water.

FIG. 157.

In Fig. 158 is shown a practical form of the pump. The water which supplies the energy for pumping enters

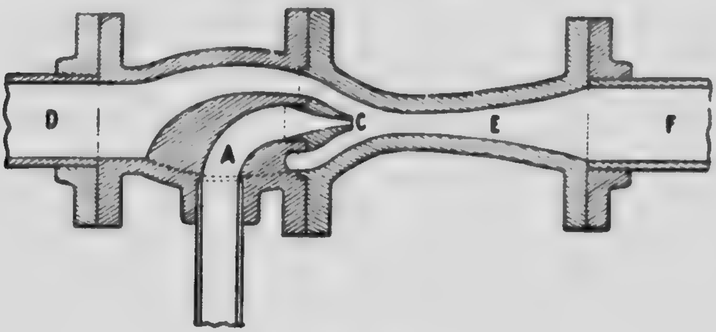

FIG. 158.

at A. It discharges at C, and the water from D is carried on by the pipe E to the pipe F.

16. The Bunsen Filter Pump.

Appliances for producing a suction current of air are known as aspirators. One of the best known of these is the Bunsen Filter Pump, a vertical section of which is shown in Fig. 159. Water is forced through the tube-nozzle N, which gradually tapers and then expands again. At the place where its section is least there is joined on an off-set tube A, which is connected to the vessel from which the air is to be removed. The water, rushing through the narrow passage, attains a great velocity; the pressure is accordingly reduced until much below atmospheric pressure and the air flows in through A and is carried off with the water.

FIG. 159.

17. The Atomiser.

The atomiser is an instrument for reducing a liquid to a fine spray. Its construction is shown in Fig. 160. On pressing the bulb B an air blast is forced in a jet from the fine opening A. It crosses the top of the tube C, and as its velocity is great the pressure just above the top of C is much reduced. The pressure of the atmosphere on the surface of the liquid D forces it up the tube, and as it escapes it is blown into a fine spray.

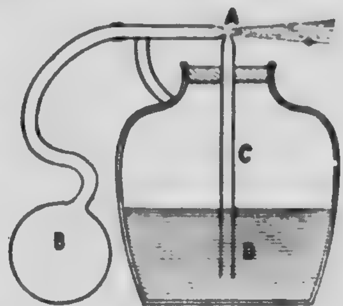

FIG. 160.

The atomiser has many practical applications. It is used to obtain a fine shower of perfume, or a fine spray of oil in oil-burning engines. Artists render permanent drawings with charcoal or crayon by spraying them with a solution of mastic in alcohol. The alcohol evaporates and leaves the picture covered with a thin transparent varnish of mastic. The atomiser is often used also in medical practice.

18. The Steam Injector.

This is an appliance for supplying steam-boilers with water, especially used with locomotives but not exclusively so. It was invented in 1858 by Giffard a French engineer. The steam and water within the boiler are under considerable pressure, but by means of the injector the steam from the boiler, or even steam at a lower pressure, is able to force water into the boiler.

In Fig. 161 is shown a longitudinal section of the injector. Steam enters at A and blows through the round orifice C. Feed water flows in at B, and meeting the steam at C, causes it to condense. In this way a vacuum is produced at C, and the water rushes in with great velocity down into the cone D, its velocity being increased by the steam from C striking it from behind. In the lower part of the nozzle E the stream expands; in doing so it loses velocity and gains pressure, and at the bottom the pressure is so great that it enters the boiler through a check valve which opens only in the direction of the stream. An overflow pipe F, by providing a channel through which steam and water may escape before the stream has acquired sufficient energy to force its way into the boiler, allows the injector to start into action. In the actual instrument there are certain valves for regulating the flow of the steam and the water which are not shown in the diagram.

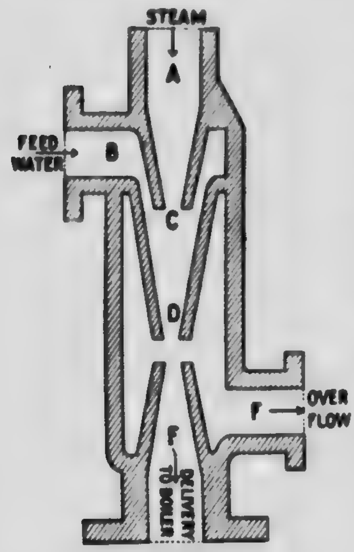

Fig. 161.

The mechanical efficiency of the injector is much lower than that of the steam pump, but it has the advantage of working when the engine is still and of heating the feed water before delivering it to the boiler.

19. The Ball Nozzle.

This is illustrated in Fig. 162. At the end of a tube is a hollow cup in which a ball fits snugly. If a vigorous current of air or steam is forced through the pipe its velocity at *a*, where it leaves the pipe is greater than at the edge of the cup where it escapes into the atmosphere. Hence the pressure at *a* is less than at the edge of the cup, and the latter is the pressure of the atmosphere. Consequently the atmospheric pressure on the side of the ball opposite *a* will prevent the ball from leaving the cup.

FIG. 162.

20. Forced Draught.

In order to keep a locomotive moving steam must be generated rapidly, and to do this a fierce fire must be maintained. To secure this the exhaust steam from the cylinders of the engine is discharged through a contracted nozzle A, a little distance below the base of the smoke-stack B, which is usually flared out like an inverted funnel (Fig. 163). The steam escapes with high velocity. This reduces the pressure greatly and produces a powerful aspiratory effect, which draws in great quantities of air through the fire-box, thus keeping up an intense fire.

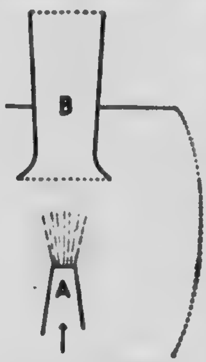

FIG. 163.

Other Illustrations of Bernoulli's Theorem.

21. Curve of a Ball.

The curve given to a ball by a 'cut' in tennis, by a 'slice' in golf or by a skilful pitcher in base-ball can also be accounted for by Bernoulli's Theorem.

In order to explain the effect it is more convenient to consider the ball as standing still while a current of air is forced past it, than to take the air as standing still and the ball to be rushing through it. From a mechanical point of view the conditions are equivalent, there is a motion of the ball relative to the air.

The essential requisite to produce a curve is to give the ball a spin as well as a motion forward. Let the ball be spinning about a horizontal axis in the direction shown by the two curved arrows (Fig. 164), and let the air current be in the direction from right to left. The ball in its spinning carries around with it some of the air near its surface. At b the air carried around by the ball will unite with the motion of the outer air current, while at a it will oppose the outer air current. Consequently the velocity of the air current at b will be greater than at a, and the pressure at a will be greater than that at b. Hence the ball will move across the air current in the direction from a to b, as shown by the arrow A. If now we consider the air to be at rest and the ball to be moving from left to right and having the same spin as before, it will curve up, as shown by B.

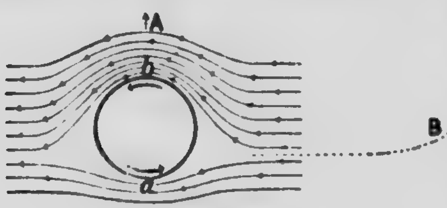

Fig. 164.

A simple way to exhibit the curved path is as follows.* Obtain a ping-pong or other light ball, varnish it, and while sticky roll it in sawdust, thus making its surface rough. After allowing to dry, place it in a pasteboard mailing tube, somewhat larger in diameter than the ball, and by a quick side-wise motion, as indicated in Fig. 165, cause the ball to roll down the tube and dart out of the end. With a little practice a decided curve can be given to the path of the ball.

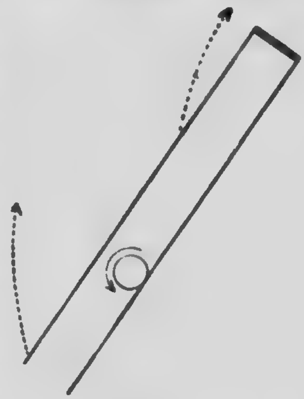

FIG. 165.

That the pressure of the air on one side of the ball is greater than that on the opposite side can be shown by

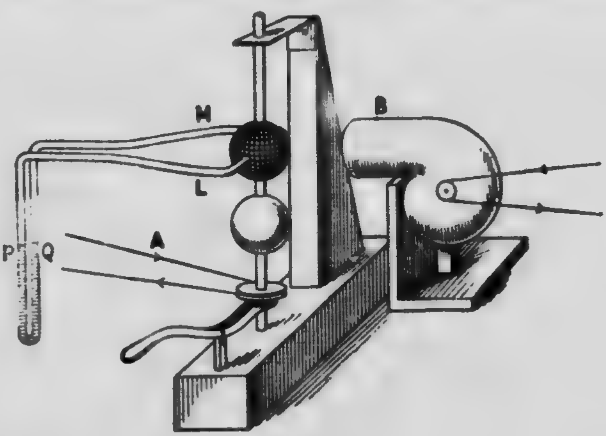

FIG. 166.

the following experiment, which is due to Sir J. J. Thomson† (Fig. 166).

*Suggested by W. S. Franklin in "Science," Dec. 15, 1911.

†"Nature," vol. 85, p. 251, 1910. (Report of a lecture before the Royal Institution, London.)

Two golf-balls, one smooth, the other with the ordinary rough surface, are mounted on an axis and can be set in rapid rotation by an electric motor. An air-blast, produced by a fan, comes through a pipe B and can be directed against the balls. By a suitable arrangement the axis can be slid up or down in its bearings so that either ball, at pleasure, may be put in the air-blast. The pressure of the air is measured by the curved tubes L, M, connected with a pressure gauge P, Q. L and M are adjusted so that the ball just fits between them.

When the ball spins in the direction indicated by the belt A the air current at L is more rapid than that at M; it is seen that the column P is depressed, Q is raised. If the smooth ball is used the effect is similar but not so pronounced. The smooth surface does not carry so much air with it as does the rough one.

FIG. 167.

22. Light Ball in a Jet of Steam or Air.

A light ball (made of celluloid, or a tennis ball), may be held in equilibrium by a jet of air or steam as illustrated in Fig. 167. The ball is under the action of three forces:—Its own weight W; I, the force of impact of the fluid against the ball; and P, an excess of atmospheric pressure over the pressure on the other side of the ball, due to the high velocity of the escaping fluid. With a few trials a position can usually be found for the ball where the three forces are in equilibrium, and the ball remains there.

23. Two Balls in a Current of Air.

If two light balls are suspended side by side in a current of air from an electric fan (Fig. 168) the wind-current between the balls is greater than that on the other side of them. The air-pressure on the outer sides is therefore greater than that in the space between, and the balls are consequently pushed toward each other.

Fig. 168.

24. Two Ships Steaming Side by Side.

If a ship is anchored in a river the water flows past it, the particles moving in definite stream lines. If the vessel is moving forward through still water, there is a similar relative motion between it and the water, and the resulting stream lines are similar to those in the other case.

If two ships are steaming side by side (Fig. 169) the water streams past them more swiftly in the space between than on the outer sides. On account of this increased velocity the pressure exerted by the water against the inner sides of the ships is less than that against the outer sides, and the ships are pushed toward each other. One might expect the water between the ships to be heaped up, but such is not the case, its level is *below* the level at other places. Large ships should not manœuvre too close to each other; accidents have occurred through ships being apparently drawn together in the manner just described.

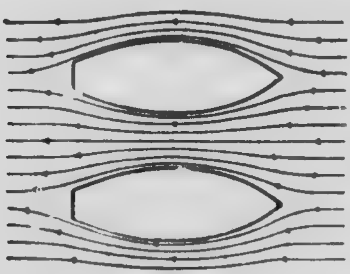

Fig. 169.

EXERCISE XLIII.

1. In a water-works system the pressure is maintained by the water in a stand-pipe 100 feet high situated on a hill 50 feet above the valley. Find the pressure, in pounds per square inch, on the ground floor of a house in the valley.

2. If the stand-pipe is 30 metres high and the hill 20 metres above the valley find the pressure in dynes per square cm., and also in kilograms per sq. cm.

3. A large tank 3 metres high is kept full by water continually running into it, and a small round opening, 1 cm. in diameter, is made at the base. At what rate will the water escape?

4. A can contains oil to a depth of 18 inches, and a small round hole $\frac{1}{16}$ inch in diameter is punched through it at the base. At what rate will the oil begin to run out? (Take velocity of efflux the same as that for water.)

5. Find the work done in pumping 20 gallons of water into a boiler in which the pressure is 50 pounds per square inch. (1 gal. = 277.3 cu. in.)

6. At what velocity must the water flow in a canal 30 feet wide and 8 feet deep to discharge 1,000 cu. ft. per second?

7. Water in a pipe is under a pressure of 60 pounds per square inch and is flowing at the rate of 5 feet per second. Find the energy per cubic inch. (Neglect potential energy due to gravity.)

8. If the pressure is 5 kilos. per sq. cm., and the rate of flow is 2 metres per second, find the energy per c.c.

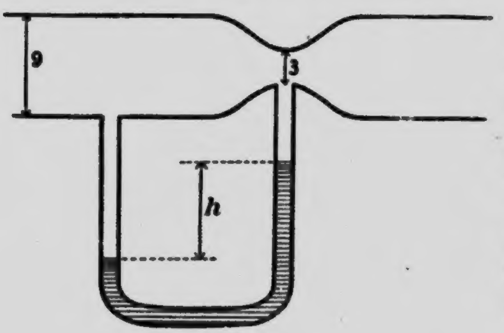

FIG. 170.

9. A water-main 7 feet in diameter has a throat inserted in it 2 ft. 4 in. in diameter. The flow of water through the pipe is 300 cu. ft. per sec., and the pressure in the 7-foot pipe is 80 pounds per square inch. Find the pressure in the throat.

10. In a glass tube 9 cm. in diameter is a throat 3 cm. in diameter, and a U tube is fused in as shown in Fig. 170. The U tube contains mercury and when the water is

flowing through the pipe it is noticed that the difference h in the mercury levels is 10 cm. Calculate the flow of water through the pipe in c.c. per sec. (Density of mercury = 13.6 grams per c.c.)

References to Works on the Flow of Liquids.

Encyclopedia Britannica, 11th Edition. Art. "Hydraulics," Vol. XIV, p. 35.

EDWIN EDSER, *General Physics for Students*, Chapters XI to XV.

FRANKLIN and MACNUTT, *The Elements of Mechanics*, Chapter X.

"School Science and Mathematics," Jan. 1911, p. 7. Article by W. S. Franklin.

W. FROUDE, "Stream Lines in Relation to the Resistance of Ships," Brit. Assoc. Report, 1875; "Nature," vol. 13, p. 50, 1875.

ANSWERS.

EXERCISE XLII.

3. $50{,}914\frac{3}{7}$ ergs ($\pi = 22/7$).
4. $2:1:\frac{1}{1.04}$, or $1:\frac{1}{2}:\frac{1}{2.08}$.
6. 3.306, 1.343, 1.376 cm., respectively.
8. Height in tube twice that between plates.

EXERCISE XLIII.

1. 65.1 pounds per sq. in.
2. 4,900,000 dynes per sq. cm.; 5 kg. per sq. cm.
3. 374.01 c.c. per sec.
4. .224 cu. in. per sec.
5. $23{,}108\frac{1}{3}$ foot-pounds.
6. $4\frac{1}{6}$ ft. per sec.
7. 5.014 ft.-pounds (nearly).
8. 4,920,000 ergs.
9. 47.1 (approx.) pounds per sq. in.
10. 3673.9 c.c. per sec.